Building Bridges – Making Connections between Counting and Arithmetic: Subitising

Judy Sayers explores subitising and whether it needs to be taught

What is subitising?

Definition: (verb) Without counting, instantly recognising the number of objects in a small group, for example, when you can see that there are five coins without counting.

Psychologists, Klein and Starkey (1988) found that subitising is innate in all of us; from the first few days of birth we are able to spontaneously recognise and discriminate small numbers of objects. For example, three pictures hang on a wall in front of a six-month-old baby (as figure 1 below). When the baby hears three drumbeats, his eyes move to the picture with three dots.

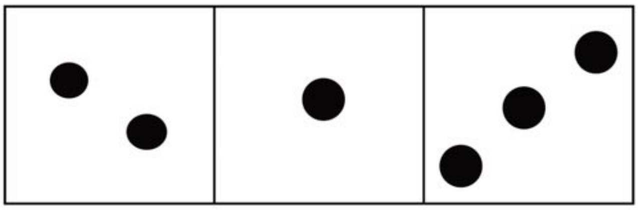

Figure 1

As a teacher, this puzzled me as some children count in ones, even when they get to key stage two. Why, if they know how to subitise at a very early age? This set me on a quest to find out more about what subitising is and if it is something we should teach.

If one reads the mathematics recovery literature (Wright et al, 2006, Munn & Reason, 2007), you will find that subitising is indeed a strategy that is taught in their intervention programmes. Although other intervention programmes conducted in England allude to the notion of subitising, it is not always made explicit.

What has been interesting, whilst researching this approach, is that some psychologists believe it is just a short cut, or rapid form of counting, but there is more to it than that. Clements' (1999) work showed that subitising can play an important role in the development of basic mathematical skills, including early arithmetic such as addition and subtraction. He also highlighted the difference between: *perceptual* and *conceptual* subitising.

Perceptual subitising is, as the definition implies above, the notion of recognising a number without using other mathematical processes, i.e. counting. It makes use of a natural mechanism similar to that used by some animals and birds. Biologically, humans are limited to how many dots they can see at once (approx. 5). It would therefore, require a conceptual understanding to know that four dots and three more make seven dots. For example, subitising more than five in a line can be very difficult, unless you provide a pattern (see figures 2 & 3 below).

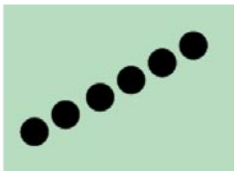

Figure 2:
6 dots to subitise

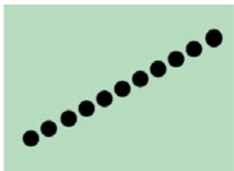

Figure 3:
11 dots are much harder to subitise

Conceptual subtising is similar to the ability of combining small sets of numbers. Patterns are integral to this ability in order to 'see' numbers in sets, e.g. the patterns on a die, dominoes and fingers, where an awareness to construct number sets and combinations of those sets can be taught. The use of such familiar combined sets could help to explain why the Maths Recovery programme has been so successful in this area, where explicit 'training' is given to children to combine sets of numbers – non-verbally. It would appear that teaching the identification of combined sets of numbers is key to developing children's numerical progression, for example, moving on thinking from counting in ones to counting in small groups. Thus, the iconic image of number quantities supports the step towards abstraction. Two subitised numbers can then be added or one taken away from the other. Making the step to using symbols is not such a big leap for children.

Indeed, as a researcher in early mathematics, one cannot help but notice the explicit emphasis made on the use of 'iconic' images in young learners' development across other European pedagogies. The use of such images has not been made explicit in the foundation stage documentation in England, or indeed key stage one in the past. However, the Department for Children Schools and Families (2009) produced a publication called: Numbers and Patterns. This booklet unraveled some of the implicit pedagogic knowledge of early number for new and practicing teachers, providing them with useful structures and ideas in presenting number and early algebra to young learners. Sadly, since the new National Curriculum (2014) does not include subitising at all, much then depends on how this concept is understood and developed practically in the classroom.

In the past, two notions have dominated examples of early calculation development: concrete and abstract examples. However, that leaves out a stage of development, as Bruner's (1966) three modes of representation show in this warranted perspective on early learning of number:

> **Enactive**. Often referred to as *concrete*, where muscle memories are continually reinforced through remembering the feel of actions. Therefore thinking is based entirely on physical actions. This mode continues later in many physical activities, such as learning to ride a bike.
>
> **Iconic**. Information is stored as sensory images, usually visual ones like pictures in the mind. This begins from 18 months of age. Some will develop an extreme form of this known as eidetic imagery (photographic memory), but they usually lose it as they grow older. Thinking is based on the use of mental images (icons), which are based on sight, but can also be hearing, smell or touch.
>
> **Symbolic**/Abstract. Refers to the ability to store things in the form of symbols. This mode is acquired around six to seven years old (corresponding to Piaget's operation stage). Representation of the world is principally through language, but also other symbolic systems such as number and music.

I first became interested in iconic imaging after researching how the Japanese develop young children's counting and number conceptual understanding. They introduce young children to the Soroban abacus very early on, where an enactive use is made in moving the discs on the abacus (see figure 4). This is then later moved to an iconic image of the abacus on paper. The image is strong, and children appear to be well rehearsed in very specific motions, so that when they come to the iconic image, they continue to 'replay' those motions in their mind until they no longer need them (scaffolding). They are continually shown the abstract symbolic image of the numbers as they work through the concrete and the iconic phases. In so doing, they are developing a familiarity before they are ever expected to need them. They are not rushed into using the symbolic form too soon.

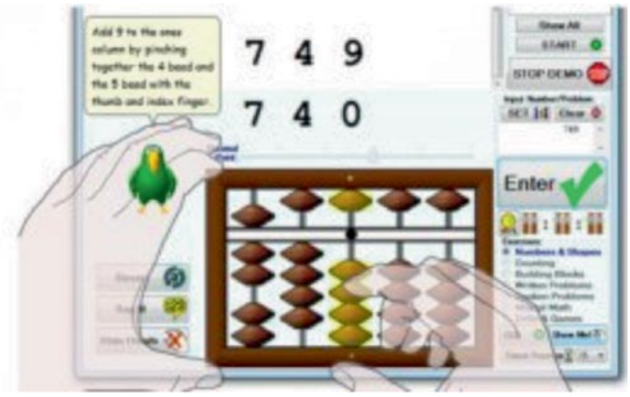

Figure 4

A similar introduction of this careful stepped process can be seen in Hungarian teaching of mathematics (Back et al, 2013). They often use a frame where iconic images are developed in the mind through the use of simple card frames of 10. Counters are used to embed the image of ten, where five becomes like the base, e.g. 6 is 5 +1, and 7 is 5 +2 etc. 17 is 5+5 (10) +5+2. See figure 5.

What has been surprising for me is that I have found a similar idea here in Sweden. They too explicitly teach children to subitise as part of those careful steps between counting and calculation (arithmetic). Their number work is very strongly supported by iconic imaging, and considering children are not placed into sets in elementary school, all children achieve well in arithmetic.

It appears then that after some simple subitising recognition of numbers, teachers can provide iconic

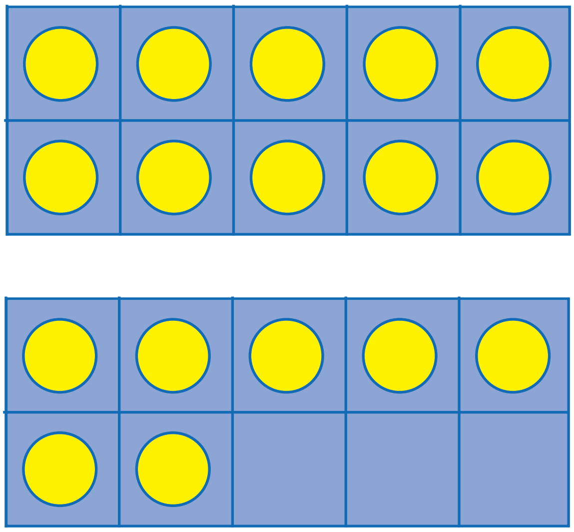

Figure 5

images of numbers in specific patterns, like the Soroban and the frames/racks provide above. These can be used for some time whist children develop confidence and images in their mind of different numbers, before moving onto abstract forms only.

Moving on to develop conceptual subitising. Later, one can move children onto different patterns and ways of seeing numbers, supporting the development of abstraction skills (e.g. figure 6). Using one colour can be helpful at first, also playing games in pairs, such as using five pennies. One child places (covered by their hand) some of the five on a tabletop, showing their partner very quickly, to see if they can subitise the number they display. When moving onto larger numbers between five and twelve, using colour to group for larger numbers at first can be helpful, (e.g. figure 7).

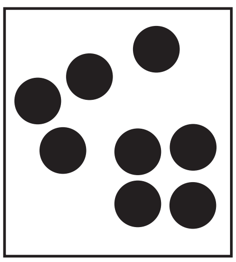

Figure 6

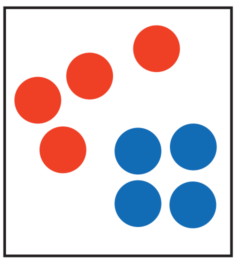

Figure 7

Larger numbers up to twenty can be explored in this random form, but essentially, the idea is to develop children's ways of seeing number sets/groups differently so that they can move from counting in ones. Using these types of activities presents confidence in number through a game. One could argue that the randomness becomes a distraction from the more regimented form of number sets like the frames seen earlier, however, evidence from Neuroscience suggests (Cutini et al., 2014) that this type of activity uses different processing skills, such as estimation.

Numicon also has qualities of imagery that supports young children's development from counting to arithmetic. Many schools I worked with in Northamptonshire have had significant improvements in overall achievement in mathematics since using those resources. However, what is important to remember is to be consistent. Numicon discuss taking a whole school approach when using their resources, but at the right time move children onto more appropriate resources to suit maturation and need (online global.oup.com, 2014). Ian Sugarman presented several iconic apparatus in *Primary Mathematics* (Autumn 2005) titled: *Beads, Racks and Counting*. Some great ideas and resources were presented to support iconic images. You will find many of these types of resources in your own classroom cupboards; if not there they are simple and inexpensive to make, or purchase. You will find resources on the web, just look for '*ten frames printable*' on a search engine, or make your own. The use of *beads* is also a helpful iconic image if used consistently (remember to use the string of beads coloured in fives e.g. five red then five white and repeated for 20).

Next step. Once children are acquainted with how to subitise, they can move onto developing concepts of number structures such as *complements* of numbers (number bonds etc.). When playing the subitising game mentioned earlier with coins, instead of subitising the amount of pennies seen, call out the complement. E.g. if three pennies of seven are shown, 'four' is shouted out. This can be done with cards too, when the numbers are increased. You will see then that it is not a great leap to using a pack of ordinary cards for this purpose. The children will begin to look only at the symbolic form of the number, not the number of spades or hearts on the cards.

The only decision you will have to make is when to move children onto more symbolic forms of calculation. Do not be afraid to let the children decide when they no longer require the iconic image, some need just a little longer than others. At this stage of their education, differences of nine months or so in age, can make a huge difference to when that decision comes.

Over to you. There are many subitising games available to download and make up for yourself.

Once you start to explore how some of the suggested resources can be used, you will begin to have other ideas of games for children to enjoy, just like I did. Use the subitising game cards from the middle pages of this journal. Use them in different ways as a game: Pairs, snap etc. with children testing each other against the clock, testing themselves etc. They can be great fun and will help children to feel more confident at this crucial step of their development of understanding in number.

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The use of the Soroban (& other Abacus models) **http://www.guardian.co.uk/science/alexs-adventures-in-numberland/2012/oct/25/abacus-number-joy-japan**

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